2:38
2:44
6 ⇒ 20 min. in class

Quiz 2, Calculus III – No Calculators

Dr. Graham-Squire, Fall 2013

Name: Key

1. (2 points) Calculate the limit: $\lim_{t\to 1}\left(\sqrt{t}\mathbf{i}+\frac{\ln t}{3t-3}\mathbf{j}+\frac{t-1}{t^2-2t+1}\mathbf{k}\right)$

$= \langle \sqrt{1}, \lim_{t\to 1}\frac{\ln t}{3(t-1)}, \lim_{t\to 1}\frac{t-1}{(t-1)(t-1)} \rangle$

L'Hospital's Rule. $= \langle 1, \lim_{t\to 1}\frac{\frac{1}{t}}{3}, \lim_{t\to 1}\frac{1}{t-1} \rangle$

$= \langle 1, \frac{1}{3}, \text{dne} \rangle$

$\Rightarrow$ limit dne

2. (4 points) Find the curvature of the plane curve $\mathbf{r}(t) = \langle e^t, 4t\rangle$ at the point (1,0). Leave your answer in exact form.

$$\kappa = \frac{|x'y'' - y'x''|}{((x')^2+(y')^2)^{3/2}}$$

$x' = e^t \quad y' = 4$

$x'' = e^t \quad y'' = 0$

$$= \frac{|e^t\cdot 0 - 4e^t|}{[e^{2t}+4^2]^{3/2}}$$

at (1,0) have $t=0$

$$= \frac{|-4|}{[17]^{3/2}} = \boxed{\frac{4}{17^{3/2}}}$$

3. (4 points) Find the indefinite integral: $\int (4\sqrt{t}\mathbf{i} + t\sin t\mathbf{j} + e^{2t}\mathbf{k}\, dt$

$= \left| 4 \cdot \frac{2}{3} t^{3/2} \vec{i} + (-t\cos t + \sin t)\vec{j} + \frac{1}{2} e^{2t} \vec{k} + \vec{C} \right.$

$u = t \quad dv = \sin t\, dt$

$du = dt \quad v = -\cos t$

$\int t \sin t\, dt$

$= -t\cos t - \int (-\cos t)\, dt$

$= -t\cos t + \sin t$